



Impulse Momentum Collisions

NJ-OER TOPIC-8



Learning Outcomes

Recognize the definition of momentum and impulse and their vector nature

Identify the mathematical quantities which effect the momentum and be able to calculate momentum from mass and velocity

Define impulse, indicate its units and relate its significance to a collision

Determine a momentum change and a total system momentum and to state what momentum conservation means

Concepts

F = Average force

I = impulse

t = impact time

v = velocity

m = mass

P = momentum

\mathbf{F} = Force as vector

\mathbf{P} = Momentum as vector

F_x = x component of the force

F_y = y component of the force

P_x = x component of the momentum

P_y = y component of the momentum

P_i = initial momentum

P_f = final momentum

P = initial momentum

P' = final momentum

Σ = sum

Units

SI Units

Force is in Newton's "N"

Energy is in Joules "J"

Impulse is in "N.s"

Momentum is in kg m/s ; no special units

Mass is in kilogram "kg"

Angle is in degrees or radian

Formulas

$$I = F t$$

$$P_{fx} - P_{ix} = F_x t$$

$$P_{fy} - P_{iy} = F_y t$$

Collisions/Systems: Conservation of momentum

$$\sum P_i = \sum P_f \quad \sum P = \sum P'$$

$$\sum P_{ix} = \sum P_{fx} \quad \sum P_x = \sum P_x'$$

$$\sum P_{iy} = \sum P_{fy} \quad \sum P_y = \sum P_y'$$

$$\sum P_i = \sum P_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_{\text{total}} v_f \quad 100\%$$

inelastic

$$\sum KE_i = \sum KE_f$$

$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

Elastic collisions only

DISCLAIMER

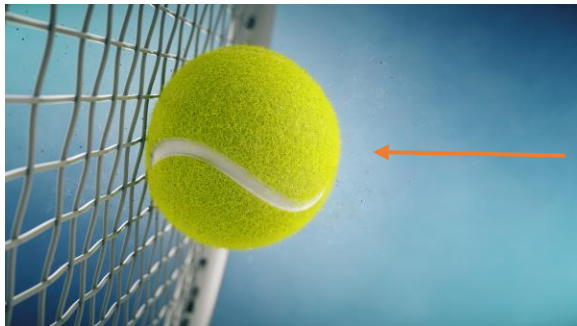
ALL ONE OBJECT PROBLEMS ARE IMPULSE PROBLEM

MOMENTUM IS NOT CONSERVED DUE TO THE EXTERNAL FORCE

1 Object 1-D Impulse

Q: A 0.060-kg tennis ball is coming to a tennis player with a speed of 42 m/s. It is struck by the racket with an average force of 77 N, which results in a speed of 28 m/s in the opposite direction from the original velocity. What is the interaction time between the ball and the racket?

Before (initial)



$$m = 0.60 \text{ kg} \quad v_i = -42 \text{ m/s}$$

$$p_i = 0.60 (-42) = -25.2 \text{ kgm/s}$$

During

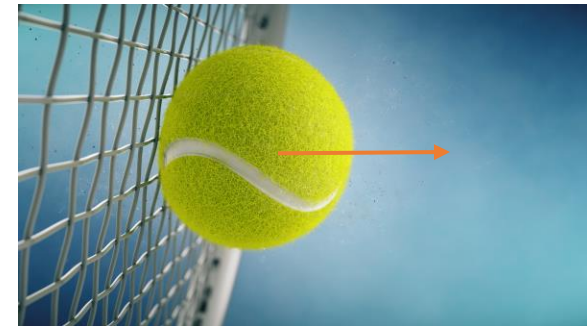


$$F = 77 \text{ N} \quad t = ?$$

$$I = 77 t$$

$$p_f - p_i = I$$

After (final)



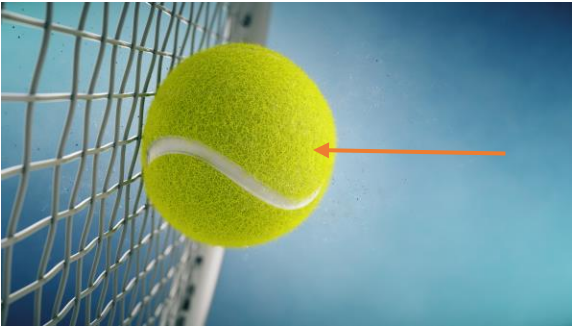
$$m = 0.60 \text{ kg} \quad v_f = 28 \text{ m/s}$$

$$p_f = 0.60 (28) = 16.8 \text{ kgm/s}$$

1 Object 1-D Impulse

Solution Numerically

Before (initial)



$m = 0.60 \text{ kg}$ $v_i = -42 \text{ m/s}$

During



$F = 77 \text{ N}$ $t = ?$

After (final)

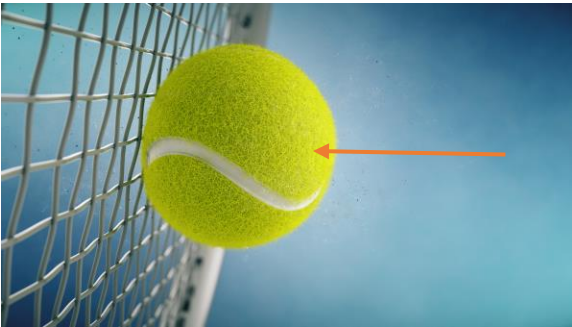


$m = 0.60 \text{ kg}$ $v_f = 28 \text{ m/s}$

1 Object 2-D Impulse

Solution Symbolically

Before (initial)



$$m = 0.60 \text{ kg} \quad v_i = -42 \text{ m/s}$$

$$P_i = m v_i$$

$$I = P_f - P_i$$

$$F t = m v_i - m v_f$$

$$t = (m v_i - m v_f) / F$$

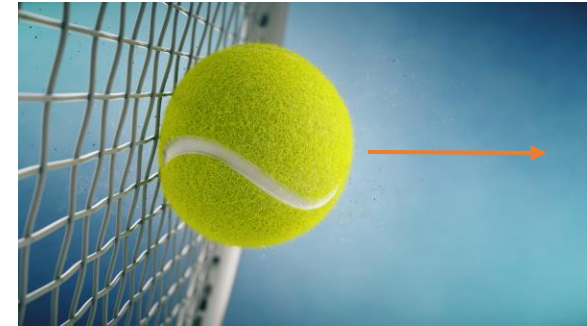
During



$$F = 77 \text{ N} \quad t = ?$$

$$I = F t$$

After (final)



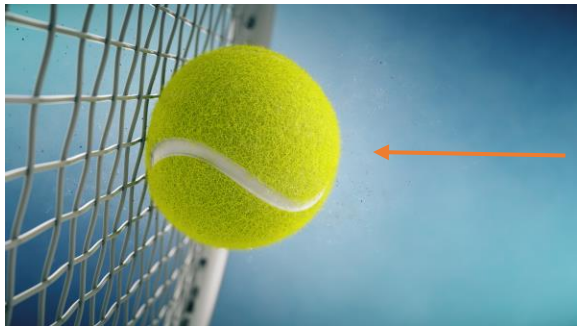
$$m = 0.60 \text{ kg} \quad v_f = 28 \text{ m/s}$$

$$P_f = m v_f$$

1 Object 2-D Impulse

Q: A 0.0600-kg tennis ball is coming to a tennis player with a speed of 42.0m/s towards West. It is struck by the racket diagonally which results in a speed of 36.0 m/s now making 30.0 degrees with the incoming direction going 30.0 degrees North of East. What was the x-component of the Force (F_x) if the interaction time between the ball and the racket is 0.400 seconds?

Before (initial)



$m = 0.60 \text{ kg}$ $v_{ix} = -42 \text{ m/s}$ $v_{iy} = 0$

During



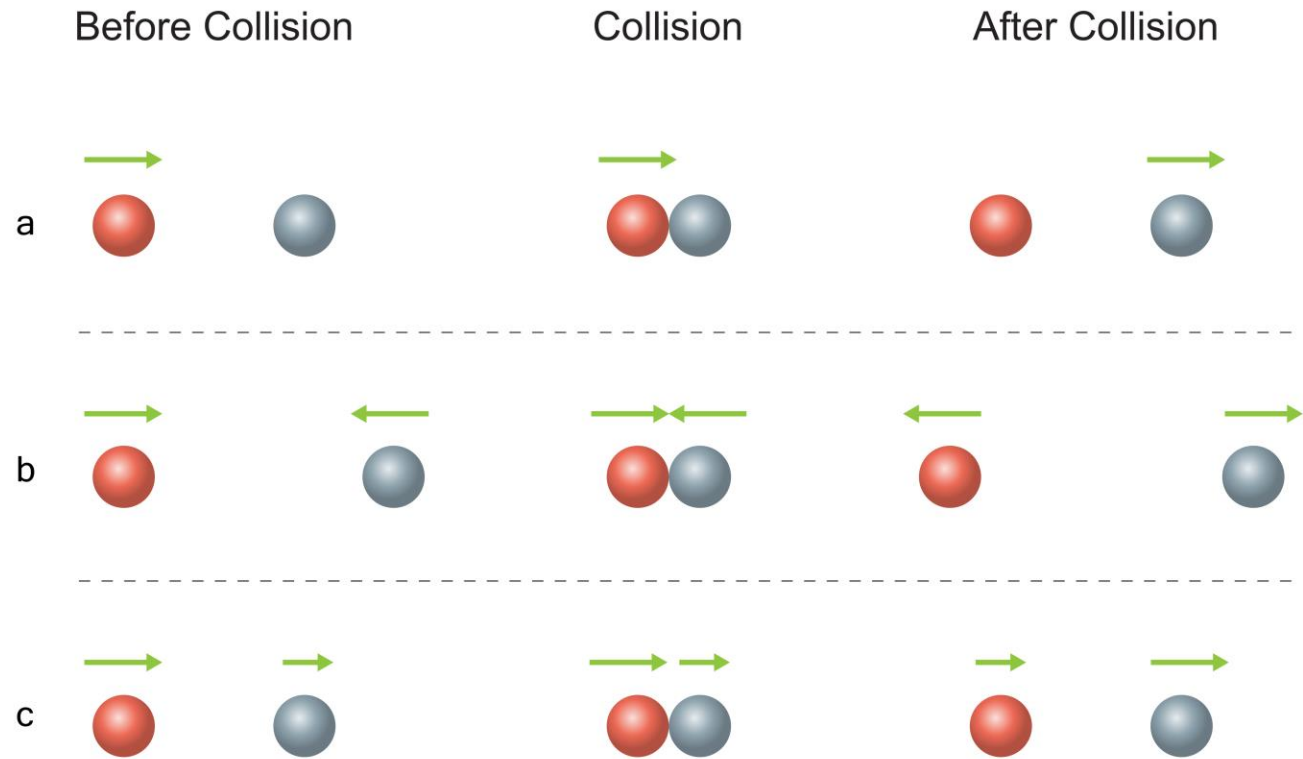
$F_x = ?$ $F_y = ?$ $t = 0.40\text{s}$

After (final)

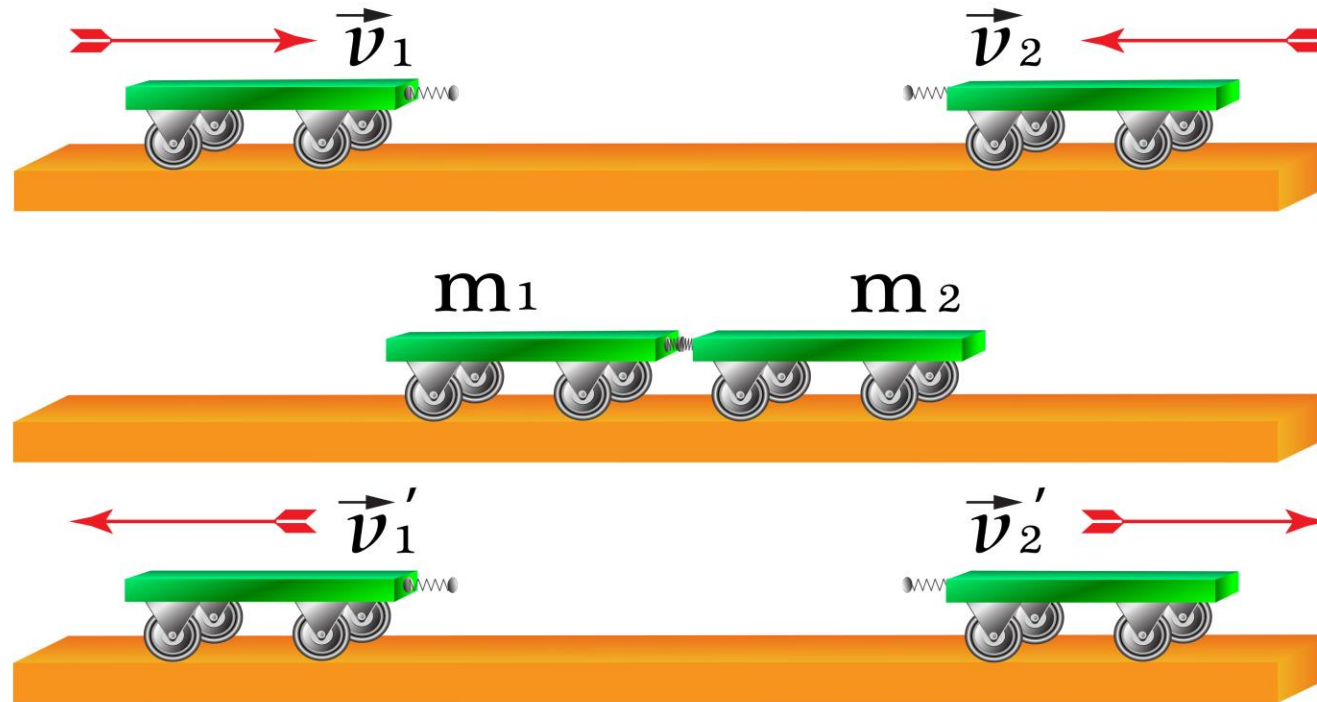


$m = 0.60 \text{ kg}$ $v_{fx} = 36 \cos(30)$ $v_{fy} = 36 \sin(30)$

TYPICAL 1-D COLLISIONS

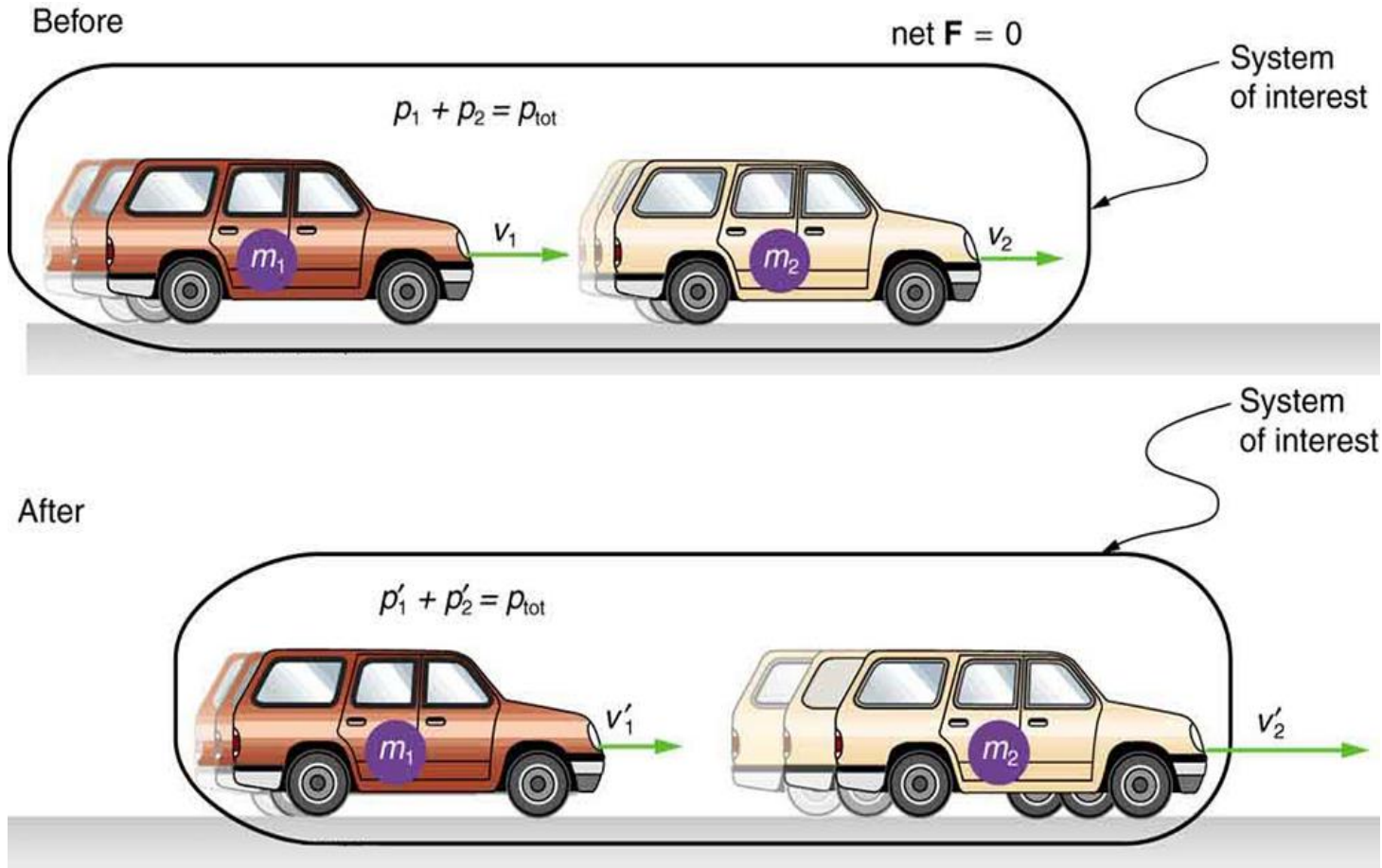


GENERAL FORMULA FOR THE COLLISIONS



$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}'_1 + m_2 \vec{v}'_2$$

1-D Collisions



Q: A 1200 kg red car moving with 18.0 m/s East hits a 1000 kg grey car moving with 12.0 m/s East. After the collision, the red car slows down to 15.0 m/s due to the impact. What is the final velocity of the grey car assuming that there is no external force

COLLISION TYPES

- 1-D Perfectly inelastic collisions
- 1-D Inelastic collisions
- 1-D Elastic collisions
- 2-D Collision (Elastic-Inelastic-Perfectly Elastic)

In all collisions total momentum is conserved and net impulse is zero if there are no external effect.

$$P_i(\text{total}) = P_f(\text{total}) \text{ or } P_{\text{total}} = P'_{\text{total}}$$

ELASTIC COLLISIONS ALSO CONSERVES ENERGY

$$E_i = E_f \text{ or } E = E' \text{ which leads to } v_{1f} + v_{1i} = v_{2f} + v_{2i} \text{ or } v_{1'} + v_1 = v_{2'} + v_2$$

FOR 2-D Collisions components of the momentum is conserved

$$P_{ix}(\text{total}) = P_{fx}(\text{total}) \text{ and } P_{iy}(\text{total}) = P_{fy}(\text{total})$$

1-D Collision with Simulation-Inelastic

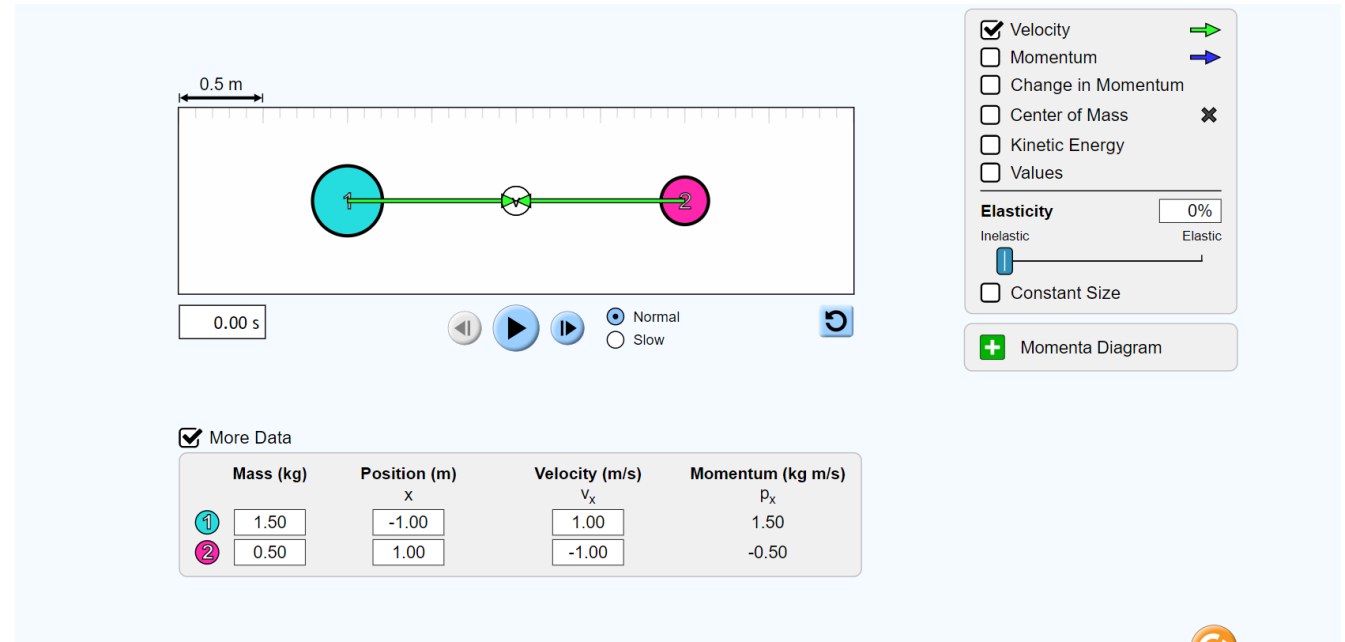
Q1) A 1.5 kg mass going East with 1.0 m/s collides with a 0.5 kg mass going West with 1.0 m/s. Find their final velocity after the perfectly inelastic collision.

Formula: $P_i(\text{total}) = m(\text{total}) V_f$

Open the link

https://phet.colorado.edu/sims/html/collision-lab/latest/collision-lab_en.html

Obtain the following image for the initial collision.



Make sure you change the values for Mass and velocity, The position is not important. Use the slider to obtain 0% elasticity. Hit the start button and verify your answer.

1-D Collision with Simulation-Inelastic

https://phet.colorado.edu/sims/html/collision-lab/latest/collision-lab_en.html

Q) Use the same link and perform the same simulation with the quantities given on the right. Use the formula below to estimate V_f .

Compare your calculations with the result of the simulation

$$P_i(\text{total}) = m(\text{total}) V_f$$

All collisions are perfectly inelastic

- a) A 0.7kg mass moving East with 2.6 m/s collides with a stationary 0.6 kg mass in a perfectly inelastic collision
- b) A 0.8 kg mass moving with 2m/s collides with a 0.4 kg mass moving in the same direction
- c) $M_1=1.5\text{kg}$ $V_{1i}=2.0$ m/s
 $M_2=2.0$ kg $V_{2i} = -1.2$ m/s
- d) $M_1=2.2$ kg $V_{1i}= 0$ m/s
 $M_2=2.0$ kg $V_{2i} = -1.2$ m/s

1-D Collision with Simulation-Elastic

Q3) A 1.5 kg mass going East with 1.0 m/s collides with a 0.5 kg mass going West with 1.0 m/s. Find their final velocity after the elastic collision. Use

$$P_i(\text{total}) = m_1 v_1' + m_2 v_2'$$

$$v_1' - v_1 = v_2' - v_2$$

This is a system of equations

v_1' and v_2' are the unknowns

Open the link

https://phet.colorado.edu/sims/html/collision-lab/latest/collision-lab_en.html

Obtain the following image for the initial collision.

The screenshot shows the PhET Collision Lab simulation interface. The main window displays two balls on a horizontal track. The left ball is cyan (mass 1.5 kg) and the right ball is pink (mass 0.5 kg). They are moving towards each other. A green arrow indicates the cyan ball's velocity is 1.00 m/s to the right, and a red arrow indicates the pink ball's velocity is -0.50 m/s to the left. The track has a 0.5 m scale bar. The time is 0.00 s. The simulation is set to 'Normal' speed. The 'Elasticity' slider is set to 100%. The 'More Data' table is visible below the simulation.

	Mass (kg)	Position (m)	Velocity (m/s)	Momentum (kg m/s)
		x	v_x	p_x
1	1.50	-1.78	1.00	1.50
2	0.50	-0.10	-0.50	-0.25

Make sure you change the values for Mass and velocity, The position is not important. Use the slider to obtain 100% elasticity. Hit the start button and verify your answer.

1-D Collision with Simulation-Inelastic

https://phet.colorado.edu/sims/html/collision-lab/latest/collision-lab_en.html

Q) Use the same link and perform the same simulation with the quantities given on the right. Use the formula below to estimate v_1' and v_2' . All collisions are elastic.

Compare your calculations with the result of the simulation. Use

$$P_i(\text{total}) = m_1 v_1' + m_2 v_2'$$

$$v_1' - v_1 = v_2' - v_2$$

- a) A 0.7kg mass moving East with 2.6 m/s collides with a stationary 0.6 kg mass in an elastic collision
- b) A 0.8 kg mass moving with 2.0 m/s collides elastically with a 0.4 kg mass moving in the same direction with 0.2 m/s
- c) $M_1=1.5\text{kg}$ $V_1=2.0$ m/s
 $M_2=2.0$ kg $V_2 = -1.2$ m/s
- d) $M_1=2.2$ kg $V_1= 0$ m/s
 $M_2=2.0$ kg $V_2 = -1.2$ m/s

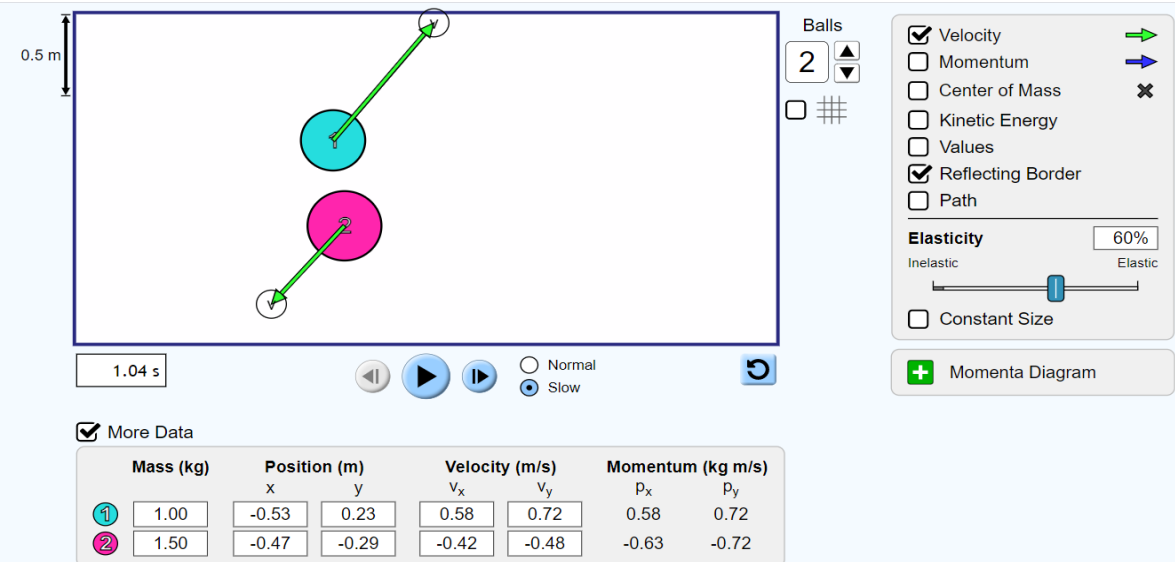
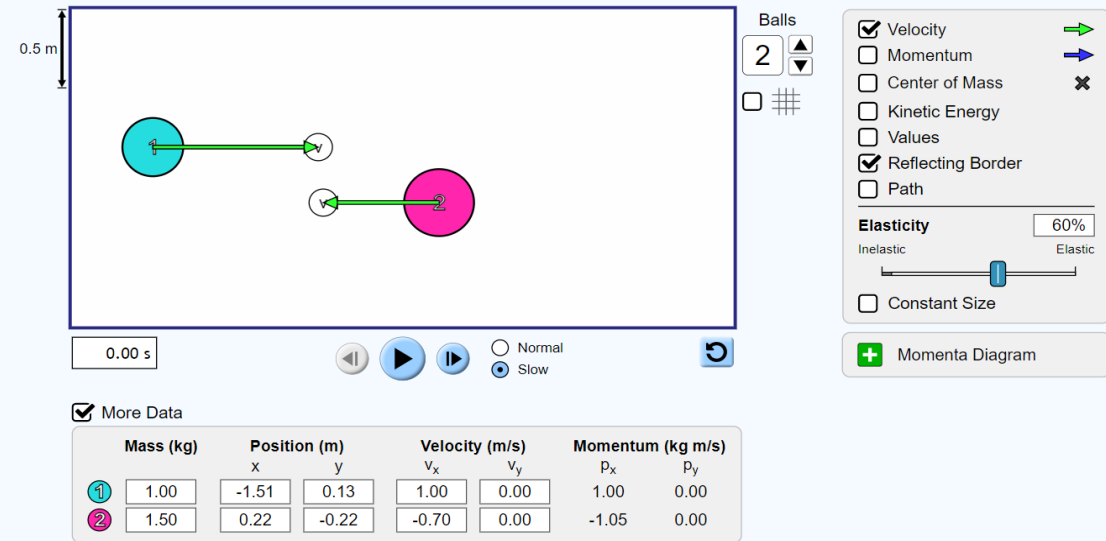
GLANCING COLLISIONS

A typical 2-D glancing collision is demonstrated on the right.

Top image is the initial and the bottom is the final.

Components of the momentum is conserved

If you know the mass, the initial velocities, and the final velocity of one of the objects, you can predict the final velocity of the second object

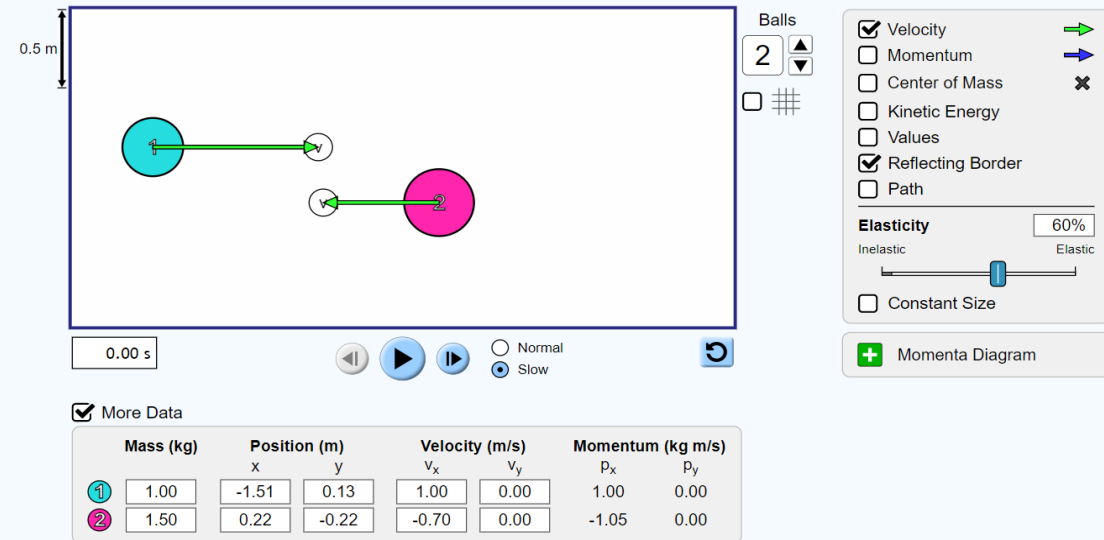


GLANCING COLLISIONS

Q5 A 1.0 kg mass going East with 1.3 m/s collides with a 1.2 kg mass going West with 1.0 m/s. After the collision 1.0 kg mass moves diagonally with a speed of 1.0 m/s making 45 degrees with the x-axis. Since $v_x = v \cos(\theta)$ $v_y = v \sin(\theta)$

Find $v_{1x'}$ and $v_{1y'}$

Calculate $v_{2x'}$ and $v_{2y'}$



$$P_x(\text{total}) = P_x'(\text{total})$$

$$P_y(\text{total}) = P_y'(\text{total})$$

$$P_x(\text{total}) = 0.1 \text{ kg m/s initial x-momentum}$$

$$P_y(\text{total}) = 0.0 \text{ kg m/s initial y-momentum}$$

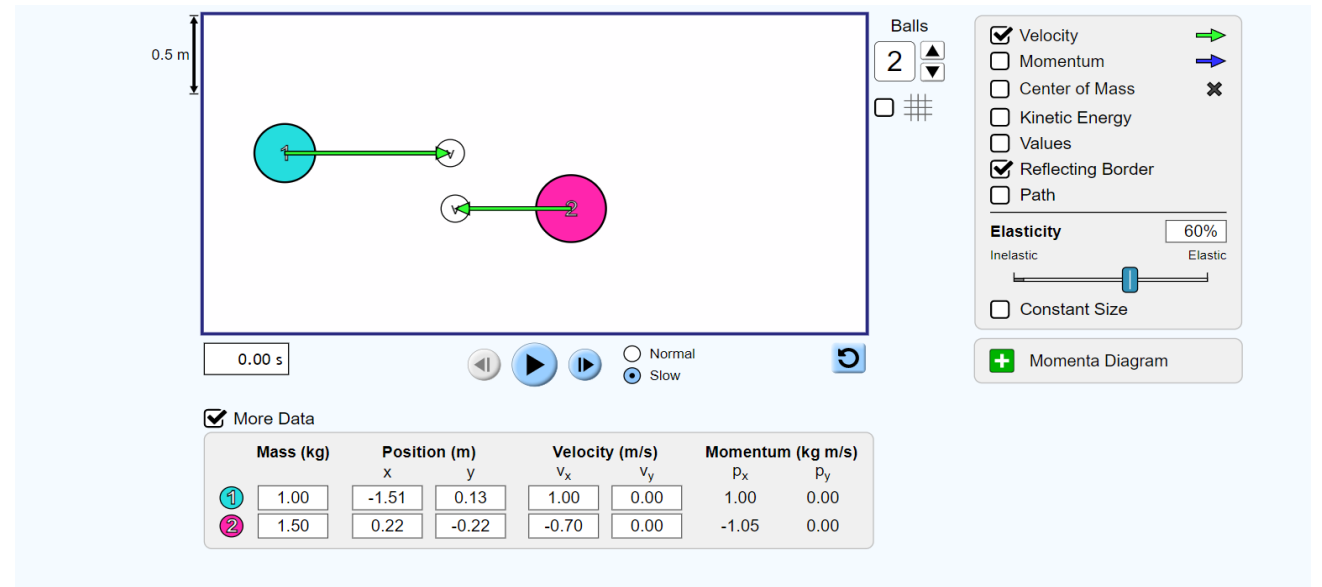
GLANCING COLLISIONS

Q A 1.0 kg mass going East with 1.3 m/s collides with a 1.2 kg mass going West with 1.0 m/s. Change the impact angle by dragging one of the masses up or down. Makes sure it results in a glancing collision.

Use the simulation and observe the final velocity of the 1.0 kg mass. Use $v_{1x'}$ and $v_{1y'}$ and the conservation of momentum, predict $v_{2x'}$ and $v_{2y'}$

Compare your results

Change the numbers and come up with your own questions.



$$P_x(\text{total}) = P_x'(\text{total})$$

$$P_y(\text{total}) = P_y'(\text{total})$$

$$P_x(\text{total}) = 0.1 \text{ kg m/s initial x-momentum}$$

$$P_y(\text{total}) = 0.0 \text{ kg m/s initial y-momentum}$$

REFERENCES

- Slide 1: Adobe id= 126428104 Balancing ball Newton's cradle pendulum By G3D Studio
- Slide 7-8-9-10: Adobe id= 245877214 Tennis racket hits tennis ball. Closeup on blue background- 3d rendering By Sashkin
- Slide 11: Adobe id= 170139266 Conservation of Momentum. Collision Before and After By fancytapis
- Slide 12: Adobe id= 251203372 Experiment - the law of conservation of the momentum of the body, the sum of the impulses of the two trolleys before the collision is equal to the momentum of these trolleys after the ... By ser68orion
- Slide 13: Open Stax College Physics online textbook
- Slide 15-17-18-19-20-21 Screenshot from PhET Interactive Simulations University of Colorado Boulder
<https://phet.colorado.edu>